Reprinted from:

JOURNAL OF APPLIED PHYSICS VOLUME 42, NUMBER 13 DECEMBER 1971

Shock-Wave Structure in Porous Solids*

Lee Davison

Sandia Laboratories, Albuquerque, New Mexico 87115

(Received 7 May 1971)

In this paper several variations of a simple theory of dynamic compaction of porous solids are presented and discussed. This theory elaborates the conventional theory of shock propagation in such a way that the shock structures observed to propagate in these materials can be described. Steady-wave profiles are calculated for several compaction models, and the inference of constitutive equations from experimental data is discussed. It is shown that the theory can be made to reproduce steady-wave profiles observed in the usual plate-impact experiments exactly.

I. INTRODUCTION

Porous solids occur in such forms as geologic materials, manufactured foams, powder-metal compacts, and ceramics. During the past decade a substantial effort has been directed toward achieving an understanding of the propagation of moderate-amplitude plane waves of uniaxial strain in these materials.[1–8] While both compressive-loading and release waves have been studied, the loading behavior has received the most attention because it is more easily investigated experimentally. From an analytical standpoint, it seems clear that the compression problem is the more difficult because unloading from partially compacted states involves only a small volume recovery along a relatively straight stress-strain path.[6,8]

For most analyses of compaction-wave propagation in these materials it has been assumed that the state of stress in the material, as it is being compacted, depends only on the state of strain. Hydrodynamic, elastoplastic, and elastic-locking models have been used extensively. In each of these models steady compaction waves are found to propagate in the form of one or more shocks. There has been some work directed toward more complicated models involving characteristic lengths[9,10] or times[11,12] but these theories are less developed. Historically, experiments have been conducted at the very high pressures induced by explosive detonation (see, for example, Refs. 13—15) and have been interpreted in terms of the Rankine-Hugoniot theory of shock propagation. The profiles of these high-amplitude waves are satisfactorily approximated as shocks because the actual wave thickness is small compared to propagation distances of interest. More recently, plate-impact experiments have been performed at pressures only moderately in excess of the static compaction threshold. The waveforms observed in these experiments are only crudely described as shocks because of the large amount of dispersion present. As an illustration of the sort of effects observed, we present the experimental records of Fig. 1(a) showing the profiles to which shocks of various amplitudes have evolved after propagating for a distance of 1.25 mm in porous iron samples. A plot of wave thickness as a function of stress amplitude is given in Fig. 1(b).

It is to describe these low-pressure observations that an improved theory must be developed. While it is easy to conceive of a number of effects that would contribute to the observed dispersion, it seems probable that the most influential is the lag experienced by the material in coming to equilibrium under load because of the time required for pore collapse. This dispersive effect is counteracted by the tendency of propagating waves to evolve toward shocks due to the rapidly decreasing compressibility of the material as it is compacted. The suggestion is obvious that here, as in the case of gas dynamics,[16] observed wave thicknesses are a result of the balance struck between these two conflicting tendencies. The fact that the stronger waves rise much more quickly than the weaker ones indicates that the shock-formation tendency is beginning to predominate over the dispersive mechanisms at the higher stresses. Unique (for a given amplitude) stable wave profiles where the tendencies are in perfect balance so that the wave can propagate unchanged in form exist and have been observed experimentally in a number of porous materials.

The objective of this paper is the exploration of a range of possible variations of a simple compaction theory, and the effect of these variations on steady waveforms. Primary attention has been given the problem of inference of constitutive equations from experimental data. Extension of the theory to cover a broader range of effects such as unloading or thermal response or treatment of the evolution of a disturbance into a steady wave is possible, but it is not discussed here. In Sec. II a brief review of some relevant aspects of wave propagation is given to introduce the notation and provide ready reference for a few formulas. The constitutive equations of a simple compaction theory are discussed in Sec. III. Section IV is devoted to the solution of specific problems, Sec. V to the experimental determination of material constitution, and Sec. VI to a summary of the important findings.

II. THEORY

A. Kinematic and Dynamic Preliminaries

In this section we consider only problems involving uniaxial compaction. The motion of a material point initially residing at a place X in an inertial coordinate space but which, at some later time t, has been

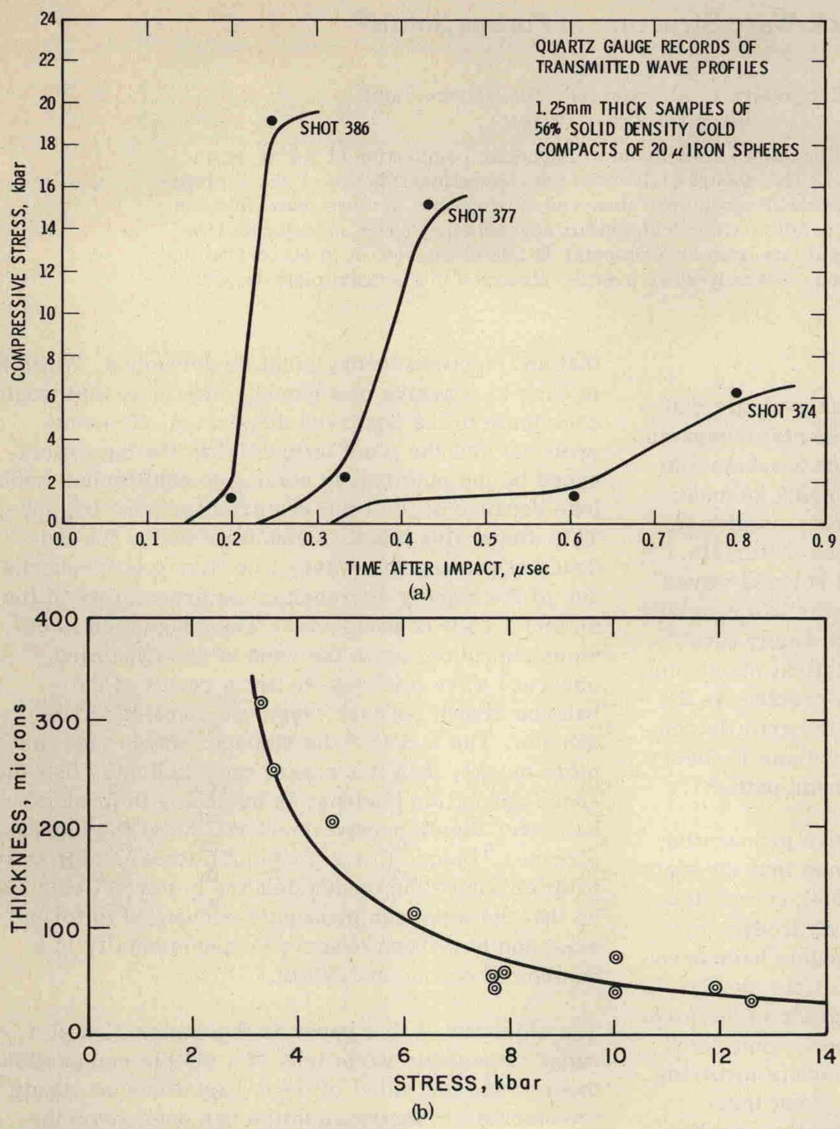

FIG. 1. Experimental results on shock compaction of porous iron. (a) shows profiles of stress waves of various amplitudes and (b) shows how the shock thickness varies with stress. These data have been communicated privately by Lysne and Halpin (Ref. 2).

displaced to a place x in this space may be expressed by an equation of the form $x=X+U(X, t)$. Strain (taken positive in compression), strain rate, and particle velocity are defined in terms of $U(X, t)$ by the relations

$$\epsilon=-U_X(X, t), \quad \dot{\epsilon}=-U_{Xt}(Xt), \quad u=U_t(x, t) \quad . \qquad (1)$$

In order that mass be conserved locally, the specific volume $v(X, t)$ of the material point originally residing at X must be related to the (constant) initial specific volume of the body by the equation $v=v_0(1-\epsilon)$. The equation of motion that must be satisfied is

$$\sigma_X+\rho_0 U_{tt}=0 \quad , \qquad (2)$$

where $\sigma(X, t)$ is the normal stress (taken positive in compression) on planes $X=$ const. This equation of motion must be augmented by constitutive equations relating the dynamic variable σ to the kinematic variable U in order that specific problems may be solved. Before addressing this aspect of the problem it is convenient to make a few general remarks on steady-wave solutions of Eq. (2).

B. Steady Waves

In this section we consider the behavior of waves that propagate at constant velocity and unchanged in form, i.e., *steady waves*.[16-19] Any traveling-wave solution $f(y\pm ct)$ of the linear-wave equation $c^2\theta_{yy}=\theta_{tt}$ has this property; this equation involves neither dispersive tendency nor tendency toward shock formation, so perfect balance is achieved in any wave and it propagates unchanged in form. As mentioned in Sec. I, it is possible that these two effects could both be present and still allow certain waves to propagate steadily because of their counterbalancing tendencies. In Sec. IV we will see that such waves do exist within the scope of the theory presented here. They have been observed experimentally and have been proven to be the stable solution to which other waveforms evolve.

To study a wave propagating at the constant velocity V we introduce the coordinates

$$\xi=X-Vt \quad \text{and} \quad \tau=t \quad . \qquad (3)$$